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OPEN Dielectric Optical-Controllable Magnifying Lens by Nonlinear **Negative Refraction**

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A simple optical lens plays an important role for exploring the microscopic world in science and technology by refracting light with tailored spatially varying refractive indices. Recent advancements in nanotechnology enable novel lenses, such as, superlens and hyperlens, with sub-wavelength resolution capabilities by specially designed materials' refractive indices with meta-materials and transformation optics. However, these artificially nano- or micro-engineered lenses usually suffer high losses from metals and are highly demanding in fabrication. Here, we experimentally demonstrate, for the first time, a nonlinear dielectric magnifying lens using negative refraction by degenerate fourwave mixing in a plano-concave glass slide, obtaining magnified images. Moreover, we transform a nonlinear flat lens into a magnifying lens by introducing transformation optics into the nonlinear regime, achieving an all-optical controllable lensing effect through nonlinear wave mixing, which may have many potential applications in microscopy and imaging science.

A traditional optical lens refracts light with designed spatially varying refractive indices to form images; such images can be magnified or demagnified according to the law of geometrical optics on both surfaces of the lens by linear refraction, e.g. plano-convex lens, plano-concave lens. These images formed by optical lenses have a limited resolution due to the well-known diffraction limit, caused by lack of detections of the near-field evanescent waves at the far field¹. In order to overcome this limit, a slab-like flat lens, namely "superlens"^{2,3}, has been demonstrated with sub-diffraction-limited resolution imaging capability in the near field by exploiting the idea of negatively refracted evanescent waves in some carefully engineered meta-materials or photonic crystals^{4,5}. However, images can only be formed by superlens in the near-field without any magnification. To mitigate these constrains, the concept of hyperlens was introduced later to convert the near-field evanescent waves into propagating ones providing magnification at the far field by the help of transformation optics^{6,7} to enable negative refraction near some hyperbolic dispersion surfaces⁸⁻¹². Besides optics, various forms of these sub-diffraction-limited resolution lenses have recently been realized in many other fields including microwave and acoustic¹³⁻¹⁵. One major drawback of these lenses is associated with high losses from metallic materials, which are the essential elements bringing in negative permittivity and artificial permeability to enable negative refraction¹⁶. Meanwhile, fabrications of such nano- or micro-structures raise additional obstacles for their practical applications.

To address this problem, alternative approaches have been proposed in nonlinear optics to achieve the nonlinear version of negative refraction using phase conjugation, time reversal and four wave mixing $(4 \text{ WM})^{17-19}$, where negative refractions can be attained by exploring nonlinear wave mixings with

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Figure 1. Illustration of the magnifying lens by nonlinear 4WM. a, Schematic of negative refraction realized by the 4WM process in a thin planar glass slide. In a special case when the pump beam at frequency ω_1 is incident normally on the glass slide, the generated 4WM beam at frequency ω_3 will be refracted negatively with respect to the angle of the probe beam at frequency ω_2 . **b**, The phase matching condition for the degenerate 4WM process in 3D wave vector space. The dashed ring line indicates the joint points of wave vector k_2 and k_3 that fulfill the phase matching condition: $2k_1 - k_2 - k_3 = 0$. **c**, Schematic of the experimental setup of the magnifying lens by 4WM. The probe beam at ω_2 that carries the object information can nonlinearly mix with the pump beam at ω_1 in a plano-concave lens to give rise to the 4WM beam which can form the magnified image of the object.

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right angle matching schemes. In contrast to those artificially engineered methods, i.e., meta-materials and photonic crystals in linear optics, ideally only a thin flat nonlinear slab is required to enable this nonlinear negative refraction²⁰⁻²⁵. Such negative refractions using nonlinear wave mixing have been demonstrated in some thin films with high nonlinearity such as the metal and graphite thin films^{20,21}. Moreover, a flat lens utilizing negative refraction of nonlinear mixing waves has successfully shown its 1D and 2D imaging ability²⁵. However, this lens still lacks the magnification capability, which is crucial for imaging applications.

Here we experimentally demonstrate a new type of dielectric magnifying lens based on nonlinear negative refraction by degenerate four-wave mixing with a thin glass slide. A multi-color imaging scheme is realized at the millimeter scale by converting the original infrared beams into the negatively refracted visible ones, spatial refractive index of the lens is carefully designed to ensure the magnification. By doing so, we surprisingly turn a demagnifying plano-concave lens in linear optics into a magnifying one in nonlinear optics. Moreover, inspired by the transformation optics, we successfully transform a non-magnified nonlinear flat lens into a magnifying one by controlling the divergence of pumping beams, effectively creating a magnifying lens controlled by another optical beam for the first time. This new imaging theme may offer a new platform for novel microscopy applications.

Results

Magnifying lens by nonlinear negative refraction. Negative Refraction can occur in a nonlinear degenerate four-wave mixing scheme^{17,19} as shown in Fig. 1a, where a thin slab of third order nonlinear susceptibility $\chi^{(3)}$ can internally mix an intense normal-incident pump beam at frequency ω_1 with an angled-incident probe beam at frequency ω_2 , generating a 4 WM wave at frequency $\omega_3 = 2\omega_1 - \omega_2$, which is negatively refracted with respect to the probe's incidence^{20,25}. Such nonlinear negative refraction arises from the momentum requirement of the phase matching condition: $k_3 = 2k_1 - k_2$ during 4 WM in order to ensure efficient wavelength conversion. This phase matching condition can be further translated to a Snell-like angle dependence law and create an effective negative refractive index n_e as (Supplementary Section 1):

$$n_e = \frac{\sin \theta_2}{\sin \theta_3} = -\frac{\lambda_2}{\lambda_3},\tag{1}$$

where the ratio of *sine* values between the probe's incident angle θ_2 and the 4WM's refraction angle θ_3 (Fig. 1a) negatively proportions to the ratio between their wavelengths λ_2 and λ_3 . The negative sign indicates reversed angles with respect to the central pump's axis, effectively creating a "negative refraction" between the probe and the 4WM wave. Meanwhile, the phase matching condition in three-dimensional wave vector space (Fig. 1b) exhibits a double cone shape around the central pump's axis, where the joint points between the incident probe's wave vector k_2 and the 4WM's wave vector k_3 compose a ring in

the transverse plane. Physically, this means that all the incident probe beams with angles parallel to k_2 , which are emitted from a point source, will be negatively refracted through 4 WM waves and focus on the other side of the slab. This builds the foundation for imaging using such negative refraction by nonlinear four-wave mixing with a thin nonlinear flat slab²⁵. Both 1D and 2D images can be obtained by a non-linear flat lens in such manner. However, due to one-to-one correspondence between the object points and the image points, the images' sizes are the same as the objects' without any magnification, similar to the case with a superlens². In order to overcome this magnification issue, a negative diverging lens, e.g., plano-concave lens, can be combined with the nonlinear negative refraction to reduce the converging angles of the 4WM beams, such that a real magnified image can be obtained, as shown in Fig. 1c. As contrast, such a plano-concave lens in linear optics only forms a demagnified virtual image with the same color; our nonlinear plano-concave lens can magnify the image with another color through nonlinear 4WM.

To elaborate this idea, we consider a four-wave mixing process in a plano-concave lens as shown in Fig. 2a. An intense normal incident pump beam can nonlinearly mix with a probe beam with an incidence angle matching the 4 WM phase matching condition in Fig. 1b to generate a 4 WM beam. In a nonlinear flat lens (i.e., double plano-surface slab)²⁵, such 4 WM beams can be negatively refracted with respect to the probe as shown as the dash lines in Fig. 2a according to the nonlinear refraction law in Equ. (1). With a plano-concave lens, this nonlinear negative refraction can be weakened by the linear Snell's refraction law on the concave surface (solid green lines in Fig. 2a), giving a magnified image. Therefore, by combining both the nonlinear refraction law and the linear Snell's law, we can obtain the magnification as (Supplementary Section 2):

$$M_1 = a \cdot \frac{v}{u} = \frac{1}{1 - \frac{u}{f} \cdot \frac{1}{a}},$$
(2)

where $a = \frac{\tan \theta_3}{\tan \theta_2}$, θ_2 and θ_3 are the probe's incident angle and 4 WM's refraction angle. *f* is the focal length of the plano-concave lens and *u* and *v* are the object distance and the image distance from the lens.

In our experiment, the pump beam with the pulse duration of ~75 fs and central wavelength $\lambda_1 = 800$ nm is delivered by a Ti:Sapphire femtosecond laser source, while another optical parametric amplifier provides pulses of similar duration at wavelength $\lambda_2 = 1300 \text{ nm}$ as the probe beam. A plano-concave lens made of BK-7 glass with focal length f = -13.5 cm, is used as our nonlinear lens, which contains the third order nonlinear susceptibility $\chi^{(3)}$ around $2.8 \times 10^{-22} \text{ m}^2/\text{V}^{226}$. The incident angle of the probe beam θ_2 is set to 7.4°, close to the phase matching condition inside BK-7 glass material in order to ensure nonlinear wave conversion about 10^{-5} efficiency. In a non-collinear configuration, a USAF resolution card is placed on the probe's path with a distance u away from the lens, while the images formed with the 4WM beams around 578 nm wavelength can be captured by a CCD camera. Figure 2b shows such images with different magnifications by varying the object distance. The measured magnification linearly proportions to the ratio between the image and object distances as shown in Fig. 2c: the linear fitting slope reads 0.473, similar to 0.468 calculated from Equ. (2). Figure 2d further proves the validity of Equ. (2) by only varying the objective distance u, showing good agreement between experimental measurements and the theory. It is also worth mentioning that the rainbow colors in the images are resulted from multicolor 4WM processes, which are enabled by the slight phase mismatching inside the nonlinear glass due to finite spectrum spreading of the incoming beams and the glass slide's thickness (Supplementary Section 4 and 7).

Figure 3 shows the 2D magnified images formed by the nonlinear magnifying lens in a non-collinear configuration. It is noticeable that the horizontal features are much clearer than the vertical ones. This is because that the incident pump and probe beams both lay on the same horizontal plane, where only one small portion of phase matching ring near the horizontal plane in Fig. 1b is exploited, giving a better phase matching to 4 WMs on that plane, while not to 4 WMs on the vertical one (Supplementary Section 3 and 4). Hence, 4 WMs can be better generated and focused in the horizontal plane, giving a finer resolution. To overcome this limitation, we implement a collinear configuration shown in Fig. 4 to access the full phase matching ring in 3D vector space in Fig. 1b (Supplementary Section 5), where a normal incident pump beam combined with probe beams scattered off the image object can fulfill the phase matching condition around the full ring geometry in 3D vector space (Fig. 1b) to generate 4 WMs. Unlike the non-collinear configuration, both vertical and horizontal lines are clear now in Fig. 4d,e with a magnification around ~1.87 given by Equ. (2).

Transforming a nonlinear flat lens into a magnifying one. Inspired by the development of transformation optics^{6,7}, we can transform a non-magnifying nonlinear flat lens²⁵ into a magnifying one by connecting the spatially varying index in a plano-concave nonlinear magnifying lens to the 4 WM phase match conditions (effective negative refractive index) in a non-magnifying nonlinear flat lens. Figure 5 illustrates this idea: with a nonlinear plano-concave magnifying lens mentioned above, the pump beam usually is normally incident to the front facet of the lens, diverged by the plano-concave lens due to linear refraction (Fig. 5a). This behavior can be mimicked by a point-like divergent pump beam passing through a flat slide (Fig. 5d). Meanwhile, 4WMs in Fig. 5d no longer fulfill the same phase matching





Figure 2. Imaging law of the nonlinear magnifying lens using negative refraction. a, Schematic of the imaging behavior of the magnifying lens. "O", "VI" and "I" stand for object, virtual image and image respectively. "u, w, v, f" are object distance, virtual image distance, image distance and focal length. "VI" represents a virtual image formed by a thin nonlinear flat lens. b, Experimental captured images with different magnifications by varying object distances "u". The images are recorded at image distances "v", where they are clearest. The scale bar is 500μ m. c, d, The magnification factor as a function of the ratio of the image distance and the object distance "v/u" and the object distance "u". The black circles are experimentally measured data. Solid red lines in c and d are theoretical curves according to Equ. (2).

uniformly along the transverse plane as in Fig. 1b due to the spatially varying incidence of the divergent pump beam, effectively experiencing spatially varying negative refractive index^{20,25} similar to the linear case of light propagation inside a gradient index (GRIN) lens transformed from a plano-convex lens. While traditional transformation optics relies on artificial meta-materials to produce spatial variations



Figure 3. Experimental 2D images formed by the magnifying lens in a non-collinear configuration. a, The non-collinear experimental setup: The pump and probe beams have the pulse duration of ~75 fs and repetition rate of 1 KHz. A delay line is added in the light path of the pump beam to ensure overlapping in time with the probe beam. A USAF resolution card, used as the "object", is placed on the probe's path, while the "image" formed with 4 WM beams can be captured by a color CCD camera. The focal length of the plano-concave lens is -13.5 cm and its edge thickness is 1 mm. b, Magnified images of the "numbers" in the USAF resolution card recorded at u = 3.25 cm, v = 15 cm in a non-collinear experimental setup. The corresponding original object images are shown in the insets. The scale bar is 500 μ m.



Figure 4. Experimental 2D images formed by the magnifying lens in a collinear configuration. a, The collinear experimental setup: The pump beam at $\lambda_1 = 800 nm$ is incident on the plano-concave lens normally, reflected by a dichroic mirror (900 nm long pass). The probe beam at $\lambda_2 = 1300 nm$ modulated by a "grating" is transformed and forms an "object" in the front of the lens by a 4f system. The focal lengths of " L_1 " and " L_2 " are 4 cm and 6 cm, respectively. The zero order diffraction beam of the grating is blocked because this beam can't fulfill phase matching. The focal length of the plano-concave lens used in this setup is -9.8 mm and its edge thickness is 1.98 mm. The "image" formed by the 4 WM beam at $\lambda_3 = 578 nm$ is recorded by a home build microscopy, made of a $40 \times$ objective lens, a 600 nm short pass filter, a lens with focal length 15 cm and a high sensitive CCD camera. **b-e**, Images of the gratings in a collinear experimental setup. **b**, Object image with horizontal lines. **c**, Object image with vertical lines. **d**, Magnified image of the object with horizontal lines. **e**, magnified image of the object with vertical lines. The scale bar is $10\mu \text{ m}$.

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Figure 5. Transforming a flat lens into a magnifying lens. a, Schematic of a nonlinear plano-concave magnifying lens: normally incident pump beams are diverged by the lens. "f" is the virtual focus of the plano-concave lens in linear optics. b,c, Magnified images of the gratings formed by the nonlinear plano-concave lens with focal length f = -13.5 cm. d, Schematic of a nonlinear magnifying flat lens: the pump beam emits from the point "F", diverged along the same paths as the former case behind the flat lens. 4 WMs can be generated in a similar manner in both cases. e,f, Magnified images of the gratings formed by a flat lens with a diverged pump beam 13.5 cm away from the lens. The scale bar is 10μ m.



Figure 6. Optical controlling a nonlinear magnifying flat lens. a,d, Schematic of a nonlinear magnifying flat lens with the pump distance $F_1 = -10 \text{ cm}$, $F_2 = -6 \text{ cm}$. b,c, Magnified images of the gratings formed by the nonlinear magnifying flat lens in **a** with magnification 1.31. e,f, Magnified images of the gratings formed by the nonlinear magnifying flat lens in **d** with magnification 1.58. The scale bar is $10 \mu \text{m}$.

to manipulate the light propagation in a linear fashion, our method here creates the first example ever using effective negative refractive index by nonlinear 4 WMs.

By considering both nonlinear 4 WMs and linear refraction of concave surfaces, we can further derivate the magnification factors as below within a paraxial approximation owing to the relative small phase matching angle \sim 7.6° between the probe and the pump (Supplementary Section 6):

$$M_2 = \frac{1}{1 - \frac{u}{F} \cdot \frac{\tan \theta_2'}{\tan \theta_3}},\tag{3}$$

where M_2 are the magnifications of a nonlinear flat lens with a divergent pumping. In Fig. 5d, θ_2' is the probe's incident angle. *F* is the distance between the pump and the lens. Technically, θ_2' is different from θ_2 in Equ. (2) which is the probe's incident angle in a nonlinear plano-concave lens in Fig. 5a, because they have to fulfill different phase matching due to the pump's incidence. In our case, this difference is only ~0.7°, which is within the allowed 4 WM angle spreading due to multicolor spectrums of pump

and probe beams and lens's thickness effect during 4 WMs explained in Supplementary Section 7. This makes the equivalence of Equ. (2) and Equ. (3) if f = F. Experimentally, we confirm this by transforming a nonlinear plano-concave lens with f = -13.5 cm to a nonlinear flat lens with a divergent pump 13.5 cm away from the lens, obtaining the 2D images with similar magnification ~1.26 in both cases as shown in Fig. 5b,c,e,f with a collinear configuration.

Optical controlling a nonlinear magnifying lens. At last, we show the most interesting feature by this transformed nonlinear lens: optical controlled magnification. Note that compared to Equ. (2), Equation (3) contains the effective focal length F, which can be tunable by tuning the divergence point of the pump beam, effectively optically controlling the nonlinear lens' focal length. By varying this effective focus, we can control the magnification of the formed images. For example, we experimentally can increase the magnification to 1.58 from 1.31 in Fig. 6b,c,e,f by decreasing F from -10 cm to -6 cm. This create the first example ever of an optical controllable lens, as all previous works involves mostly with liquid crystal, thermal effect or deformed liquid lenses^{27–29}, which could have slow responsibility. Such optical controllable devices may trigger new applications in imaging science.

Conclusion

In summary, we have experimentally demonstrated a dielectric nonlinear magnifying lens by nonlinear refraction through four-wave mixing in a thin glass slide. Our method explores the possibility of using dielectric's nonlinear properties for negative refraction as a substitute approach for meta-materials to overcome the loss problem. We extend the transformation optics into nonlinear regime, creating a non-linear optical-controlled magnifying lens. The new nonlinear optical lens design reported here may open new realms of many applications in microscopy and imaging science in the near future.

References

- 1. Pendry, J. B. Negative refraction makes a perfect lens. Phys. Rev. Lett. 85, 3966-3969 (2000).
- 2. Fang, N., Lee, H., Sun, C. & Zhang, X. Sub-diffraction-limited optical imaging with a silver superlens. *Science* **308**, 534–537 (2005).
- 3. Parimi, P. V., Lu, W. T. T., Vodo, P. & Sridhar, S. Photonic crystals Imaging by flat lens using negative refraction. *Nature* 426, 404–404 (2003).
- 4. Pendry, J. B., Schurig, D. & Smith, D. R. Controlling electromagnetic fields. Science 312, 1780-1782 (2006).
- 5. Pendry, J. B. & Smith, D. R. Reversing light with negative refraction. Phys. Today 57, 37-43 (2004).
- 6. Chen, H. Y., Chan, C. T. & Sheng, P. Transformation optics and metamaterials. Nat. Mater. 9, 387-396 (2010).
- 7. Lai, Y. et al. Illusion Optics: The Optical Transformation of an Object into Another Object. Phys. Rev. Lett. 102, 253902 (2009).
- 8. Liu, Z. W., Lee, H., Xiong, Y., Sun, C. & Zhang, X. Far-field optical hyperlens magnifying sub-diffraction-limited objects. *Science* **315**, 1686–1686 (2007).
- 9. Jacob, Z., Alekseyev, L. V. & Narimanov, E. Optical hyperlens: Far-field imaging beyond the diffraction limit. *Opt. Express* 14, 8247–8256 (2006).
- 10. Smolyaninov, I. I., Hung, Y. J. & Davis, C. C. Magnifying superlens in the visible frequency range. Science 315, 1699-1701 (2007).
- 11. Rho, J. et al. Spherical hyperlens for two-dimensional sub-diffractional imaging at visible frequencies. Nat. Commun. 1, 143 (2010).
- 12. Lu, D. L. & Liu, Z. W. Hyperlenses and metalenses for far-field super-resolution imaging. Nat. Commun. 3, 1205 (2012).
- 13. Kundtz, N. & Smith, D. R. Extreme-angle broadband metamaterial lens. Nat. Mater. 9, 129-132 (2010).
- 14. Grbic, A. & Eleftheriades, G. V. Overcoming the diffraction limit with a planar left-handed transmission-line lens. *Phys. Rev. Lett.* 92, 117403 (2004).
- 15. Li, J. S., Fok, L., Yin, X. B., Bartal, G. & Zhang, X. Experimental demonstration of an acoustic magnifying hyperlens. *Nat. Mater.* 8, 931–934 (2009).
- 16. Veselago, V. G. The electrodynamics of substances with simultaneously negative values of \in and μ . Sov. Phys. Usp. 10, 509–514 (1968).
- 17. Pendry, J. B. Time reversal and negative refraction. Science 322, 71-73 (2008).
- 18. Maslovski, S. & Tretyakov, S. Phase conjugation and perfect lensing. J. Appl. Phys. 94, 4241-4243 (2003).
- 19. Aubry, A. & Pendry, J. B. Mimicking a negative refractive slab by combining two phase conjugators. J. Opt. Soc. Am. B 27, 72–84 (2010).
- 20. Palomba, S. et al. Optical negative refraction by four-wave mixing in thin metallic nanostructures. Nat. Mater. 11, 34-38 (2011).
- Harutyunyan, H., Beams, R. & Novotny, L. Controllable optical negative refraction and phase conjugation in graphite thin films. *Nat. Phys.* 9, 423–425 (2013).
- 22. Renger, J., Quidant, R., van Hulst, N., Palomba, S. & Novotny, L. Free-Space Excitation of Propagating Surface Plasmon Polaritons by Nonlinear Four-Wave Mixing. *Phys. Rev. Lett.* **103**, 266802 (2009).
- 23. Renger, J., Quidant, R., Van Hulst, N. & Novotny, L. Surface-enhanced nonlinear four-wave mixing. *Phys. Rev. Lett.* **104**, 46803 (2010).
- Harutyunyan, H., Palomba, S., Renger, J., Quidant, R. & Novotny, L. Nonlinear Dark-Field Microscopy. Nano lett. 10, 5076–5079 (2010).
- Cao, J., Zheng, Y., Feng, Y., Chen, X. & Wan, W. Metal-Free Flat Lens Using Negative Refraction by Nonlinear Four-wave Mixing. *Phys. Rev. Lett.* 113, 217401 (2014).
- 26. Boyd, R. W. Nonlinear Optics, Third Edition (Academic Press, 2008).
- 27. Kuiper, S. & Hendriks, B. H. W. Variable-focus liquid lens for miniature cameras. Appl. Phys. Lett. 85, 1128–1130 (2004).
- Dong, L., Agarwal, A. K., Beebe, D. J. & Jiang, H. R. Adaptive liquid microlenses activated by stimuli-responsive hydrogels. *Nature* 442, 551–554 (2006).
- 29. Ono, H. & Harato, Y. All-optical focal length converter using large optical nonlinearity in guest-host liquid crystals. *Appl. Phys. Lett.* **74**, 3429–3431 (1999).

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Author Contributions

W.W. designed the study. W.W., X.L. and X.C. supervised the study; J.C. designed experiments setup, performed research. J.C. and C. Shang analyzed the data; W.W. and J.C. wrote the paper; Y.Z. and Y.F. provided advice and helpful theoretical discussion. All authors reviewed the manuscript.

Additional Information

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